

# Playsheet 3

## Formal Logic

MATH 130

**Directions:** Groups should consist of three or four people. Work together on each problem; do not delegate different problems to different people. Submit one **neatly written** write-up per group on the due date, and make sure all group members' names appear on the submission. Use complete sentences and **explain your reasoning**.

On this Playsheet, we'll work with truth tables and formal logic.

- **Complement:** The **complement** of a set  $S$  is the set  $S^C$  of all elements under consideration that are **not** in  $S$ .
- **Equality:** Two sets  $A$  and  $B$  are **equal** if they are subsets of each other. Put another way: they have exactly the same elements.
- **Negation:** The **negation** of a statement  $p$  is the statement  $\sim p$  ("not  $p$ ") that always has opposite truth value from  $p$ .
- **Logical equivalence:** Two statements  $p$  and  $q$  are **logically equivalent** if each implies the other. (That is, both "if  $p$ , then  $q$ " and "if  $q$ , then  $p$ " are true.) Put another way: in a truth table, their truth values are the same in all cases.

1. In the set  $\{1, 2, 3, 4, 5, 6\}$ , what is the complement of  $\{1, 3, 6\}$ ?
2. In the set of integers, what is the complement of the even integers?
3. In the set of real numbers, what is the complement of the even integers?
4. What is the negation of the statement, "Argent likes cats and dogs"?
5. Let  $p$  be the statement, "Ellie likes cats," and let  $q$  be the statement, "Ellie likes dogs."
  - (a) Express the statement, "Ellie likes cats and dogs" symbolically.
  - (b) Express the statement, "Ellie likes cats but not dogs" symbolically.
  - (c) Create a truth table to determine the possible truth values of "Ellie likes cats but not dogs."
  - (d) In your truth table, negate that statement.
  - (e) Translate the negation of the statement "Ellie likes cats but not dogs" from your truth table back into English.
6. Create a truth table for "if  $p$  then  $(q$  or  $r)$ ."
7. Make a truth table for "(if  $p$  then  $q$ ) or  $r$ ."
8. Make a truth table for "if  $p$  then  $q$ " and "if  $q$  then  $p$ ." Compare: are these logically equivalent?
9. When my daughter Ellie was little, whenever she ate pineapple, she would get a rash around her mouth.

- (a) Express “whenever she ate pineapple, she would get a rash around her mouth” as an if-then statement.
- (b) One day, I found her with a rash around her mouth. What, if anything, could I logically conclude from that?